

# Online Pricing for Multi-User Multi-Item Markets

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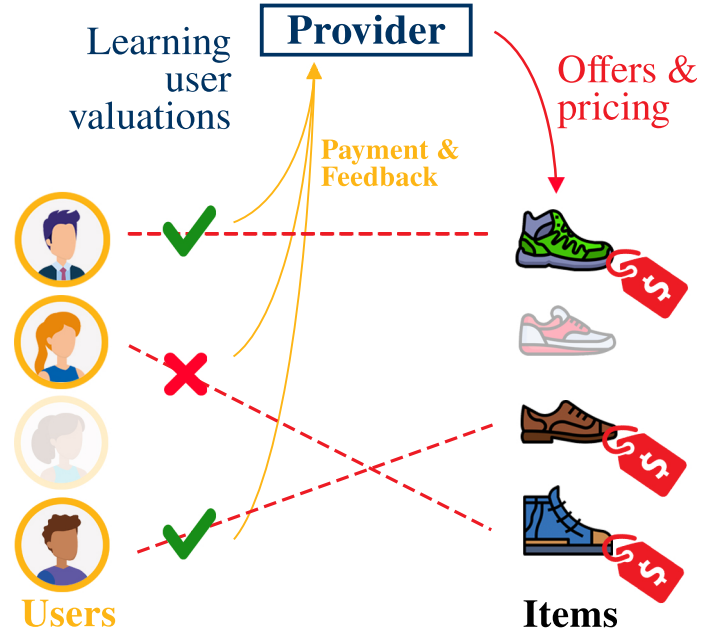
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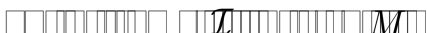
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# Introduction

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- **Learning** 
- **Accept/reject** 
- **Goal:** 
-  offers  pricing 

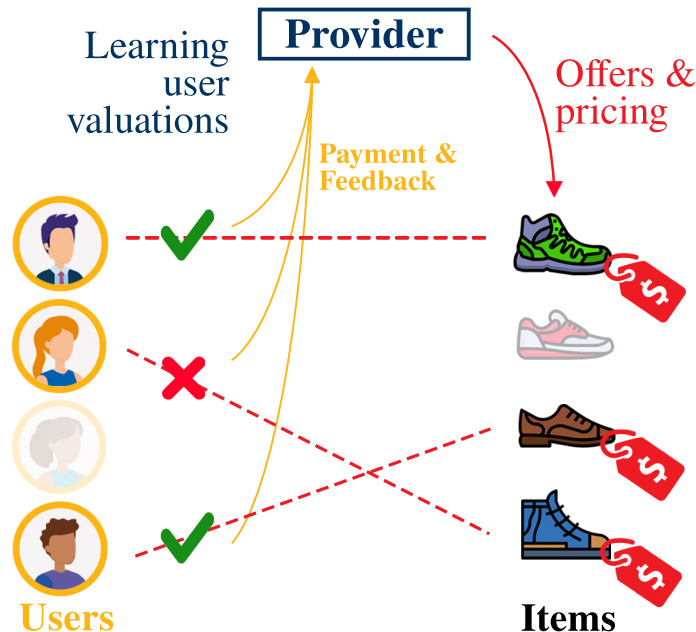


# Problem Setting

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At each round  $t \in 1, \dots, T$ :

- A subset of users  $\mathcal{D}^t \subseteq \mathcal{N}$  are active.
- A subset of items  $\mathcal{E}^t \subseteq \mathcal{I}$  are available for sale.
- Each user  $u$  has a different valuation  $v_{ui}^t$  for each item  $i$ .
- The provider decides on  
offers  $\mathbf{X}^t \in \{0, 1\}^{N \times M}$  and prices  $\mathbf{p}^t \in \mathbb{R}_+^M$ .
- User  $u$  accepts an offered item  $i$  if and only if  
$$v_{ui}^t \geq p_i^t.$$



# Goal

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$$\mathcal{R}(T, \boldsymbol{\pi}) = \text{OPT} - \sum_{t=1}^T \sum_{u \in \mathcal{D}^t} \sum_{i \in \mathcal{E}^t} x_{ui}^t p_i^t \mathbb{1}\{v_{ui}^t \geq p_i^t\}$$

- OPT *denotes the revenue of the optimal algorithm (separately for each different valuation model).*

# Main Theoretical Results

- $\mathbb{R}^n$  is a Hilbert space
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# Main Theoretical Results

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- Our algorithms are optimal up to logarithmic factors!**

| Model              | Regret Upper Bound       | Regret Lower Bound            |
|--------------------|--------------------------|-------------------------------|
| Fixed Valuations   | $O(NM \log \log(LT))$    | $\Omega(NM \log \log(LT/NM))$ |
| Random Experiences | $\tilde{O}(\sqrt{NMLT})$ | $\Omega(\sqrt{NMLT})$         |
| Random Valuations  | $\tilde{O}(\sqrt{NMLT})$ | $\Omega(\sqrt{NMLT})$         |

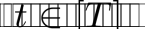
- Load parameter:**  $L = \max_{1 \leq t \leq T} \{\min(|\mathcal{D}^t|, |\mathcal{E}^t|)\}$



# Model 1: Fixed Valuations

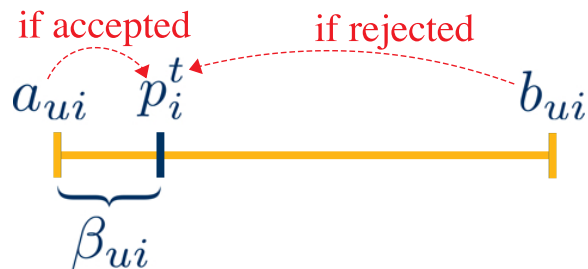
-  **do not change over time** 
-    $v_{ui}$  

$$v_{ui}^t = v_{ui}$$

  $t \in [T]$  

# Model 1: Fixed Valuations

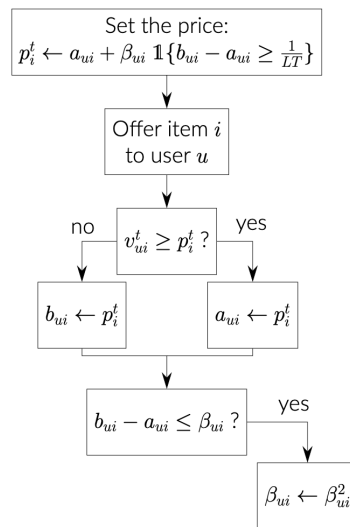
## Algorithm



Initialize the interval  $[a_{ui}, b_{ui}] = [0, 1]$  for each user-item pair.  
Initialize the search step  $\beta_{ui} = 1/2$  for each user-item pair.

At each round  $t$ :

1. Choose  $\mathbf{X}^t$  as the optimum offers under valuations  $b_{ui}$ .
2. For each offer  $(u, i)$  in  $\mathbf{X}^t$  do:



# Model 2: Random Experiences

-  **average historical experience.**

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$$v_{ui}^t = \text{avg}\{z_{ui}^\tau | \tau < t, x_{ui}^\tau = 1, v_{ui}^\tau \geq p_i^\tau\}$$

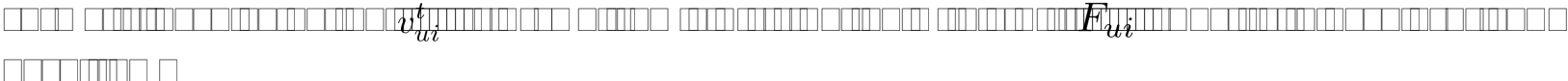
  $t \in [T]$

-   $z_{ui}^t$



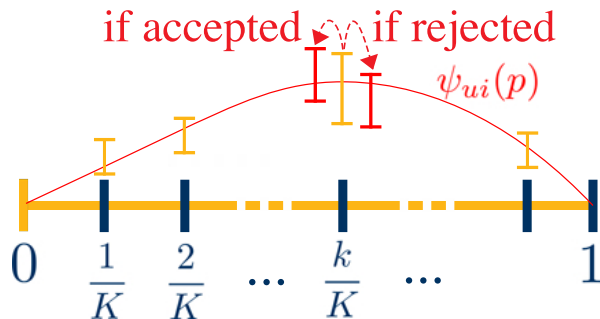
# Model 3: Random Valuations

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**independently drawn.**
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# Model 3: Random Valuations

## Algorithm



Set  $K = (LT / (NM \log(LT)))^{1/4}$ .

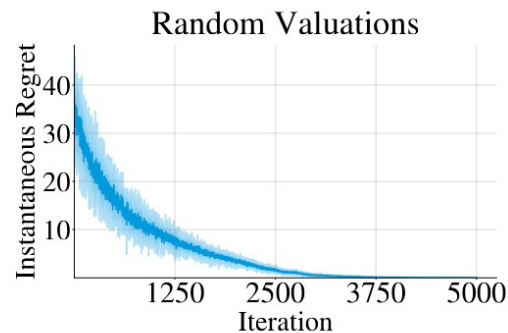
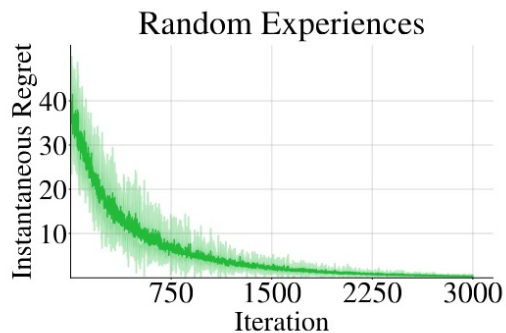
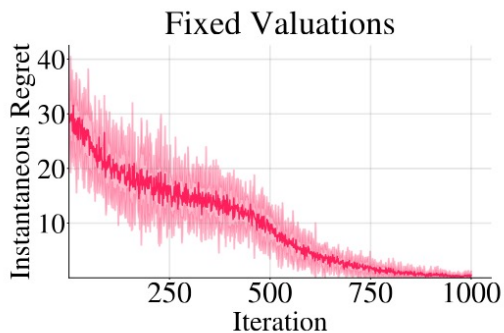
Quantize prices into  $\{1/K, 2/K, \dots, 1\}$ .

**At each round  $t$ :**

1. Set  $b_{uik} = \psi_{uik} + \sqrt{\frac{8 \log(NMKt)}{n_{uik}}}$
2. Find  $b_{ui} = \max_k b_{uik}$ .
3. Choose  $\mathbf{X}^t$  as the optimum offers under valuations  $b_{ui}$ .
4. For each offer  $(u, i)$  in  $\mathbf{X}^t$  do:
  - Set  $k^* = \arg \max_k b_{uik}$ .
  - Offer item  $i$  to user  $u$  at price  $k^*/K$ .
  - Update  $\psi_{uik^*} \leftarrow \frac{n_{uik^*} \psi_{uik^*} + p_{ui} \mathbb{1}\{v_{ui}^t \geq p_{ui}\}}{n_{uik^*} + 1}$ .

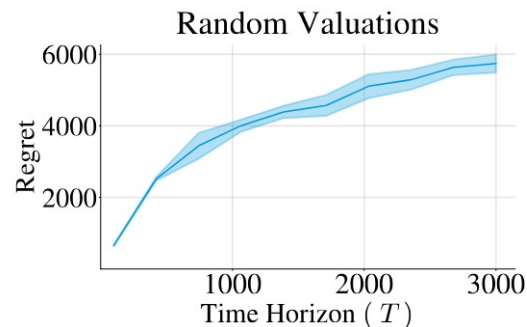
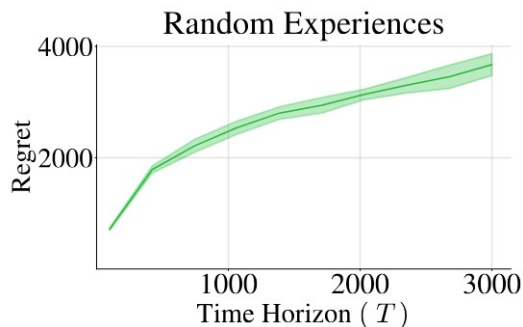
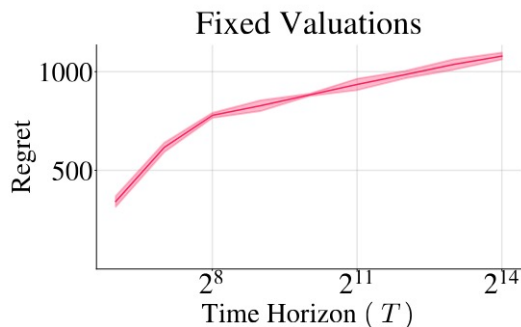
# Numerical Experiments

- **Instantaneous Regret** under different valuation models.



# Numerical Experiments

- **Regret vs. Time Horizon** under different valuation models.



- Numerical results verify that our algorithms can achieve
  - sub-logarithmic regret under the fixed valuations model,
  - sub-linear regret under random experiences model,
  - sub-linear regret under random valuations model.